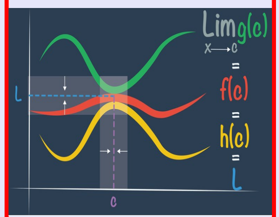
Calculus I

Lecture 42



Feb 19-8:47 AM

Class QZ 15

Given

$f(x) = x(x-4)^3$

$f'(x) = 4(x-1)(x-4)^2$

$f''(x) = 12(x-2)(x-4)$

1) Find all intercepts

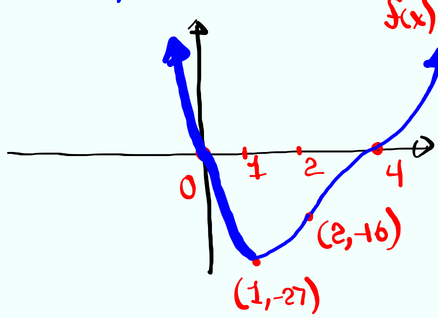
x-Int $(0,0), (4,0)$

Y-Int $(0,0)$

2) Do the Sign chart

x	$-\infty$	1		2		4	∞
$f'(x)$	$-$	•	$+$		$+$	•	$+$
$f''(x)$	$+$		$+$	•	$-$	•	$+$
$f(x)$		$(1,-27)$		$(2,-16)$		$(4,0)$	

3) Graph $f(x)$



Nov 14-7:23 AM

Given $f(x) = \dfrac{x^2}{x^2+3}$

1) Domain $x^2+3 \neq 0$

$(-\infty, \infty)$

2) All asymptotes

NO V.A.

H.A. $y = 1$ $\quad \displaystyle\lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{x^2}{x^2+3} = 1$

3) All intercepts

x-Int $(0,0)$ Twice

Y-Int $(0,0)$

4) Even, odd, or neither?

$f(-x) = \dfrac{(-x)^2}{(-x)^2+3} = \dfrac{x^2}{x^2+3} = f(x)$

Even Function

Symmetric with respect to Y-axis.

5) Find all Critical pts.

$f'(x) = 0$ or undefined

$f'(x) = \dfrac{2x(x^2+3) - x^2 \cdot 2x}{(x^2+3)^2} \qquad f'(x) = \dfrac{6x}{(x^2+3)^2}$

$(0,0)$

5) Find all possible inflection pts.

$f''(x) = 0$ or undefined.

$f'(x) = 6x(x^2+3)^{-2}$

Correction

$f''(x) = 6\left[1(x^2+3)^{-2} + x \cdot -2(x^2+3)^{-3} \cdot 2x\right]$

$= 6\left[(x^2+3)^{-2} - 4x^2(x^2+3)^{-3}\right] = 6(x^2+3)^{-3}\left[(x^2+3)^{1} - 4x^2\right]$

$f''(x) = \dfrac{6(3-3x^2)}{(x^2+3)^3} \qquad f''(x) = 0 \qquad 3 - 3x^2 = 0$

$x^2 = 1 \quad x = \pm 1$

P.I.P.

$\underbrace{\left(\pm 1, \tfrac{1}{4}\right)}_{\text{Make the Correction}}$

$\dfrac{1}{1+3} = \dfrac{1}{4}$

Set-up the Sign chart

Discuss increasing/Decreasing

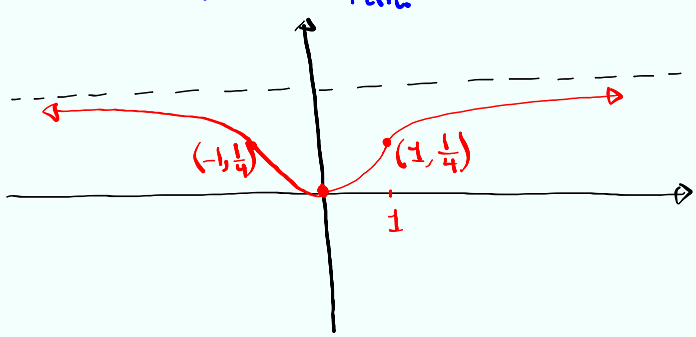
Discuss Concavity

Draw $f(x)$.

Nov 13-7:26 AM

$f'(x) = \dfrac{6x}{(x^2+3)^2} \qquad f''(x) = \dfrac{-18(x^2-1)}{(x^2+3)^3} = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$

x	$-\infty$	-1		0		1		∞
$f'(x)$	$-$		$-$	•	$+$		$+$	
$f''(x)$	$-$	•	$+$		$+$	•	$-$	
$f(x)$	↘	I.P.	↘	Min	↗	I.P.	↗	



Nov 14-7:54 AM

I have a piece of wire 10 m long.

I like to cut it and make a square and equilateral triangle.

How do I do this to get max. enclosed area or minimum enclosed area.

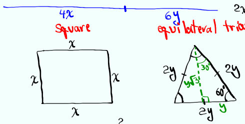
10 m

4x | 6y

$4x + 6y = 10$

$2x + 3y = 5$

$y = \frac{5-2x}{3}$



$A_{square} = x^2$

$A_{Triangle} = \frac{2y \cdot y\sqrt{3}}{2} = y^2\sqrt{3}$

Total enclosed area

$x^2 + y^2\sqrt{3}$

$$\boxed{f(x) = x^2 + \sqrt{3}\left(\frac{5-2x}{3}\right)^2}$$

$f'(x) = 2x + \sqrt{3} \cdot 2\left(\frac{5-2x}{3}\right)^1 \cdot \frac{-2}{3}$

$f'(x) = 2x - \frac{4\sqrt{3}}{3}\left(\frac{5-2x}{3}\right)$

$f''(x) = 2 - \frac{4\sqrt{3}}{3} \cdot \frac{-2}{3} = 2 + \frac{8\sqrt{3}}{9} > 0$

$f'(x) = 0$

$2x - \frac{4\sqrt{3}}{3} \cdot \frac{5}{3} - \frac{4\sqrt{3}}{3} \cdot \frac{-2}{3} x = 0$

LCD = 9

$18x - 20\sqrt{3} + 8\sqrt{3}x = 0$

$x(18 + 8\sqrt{3}) = 20\sqrt{3}$

$x = \frac{20\sqrt{3}}{18 + 8\sqrt{3}}$

$x = \frac{20\sqrt{3}}{2(9+4\sqrt{3})}$

$= \frac{10\sqrt{3}}{9+4\sqrt{3}} \cdot \frac{9-4\sqrt{3}}{9-4\sqrt{3}}$

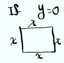
$x = \frac{90\sqrt{3} - 120}{9^2 - (4\sqrt{3})^2}$

$x = \frac{30(3\sqrt{3}-4)}{81-48}$

$x = \frac{\cancel{30}^{10}(3\sqrt{3}-4)}{\cancel{33}_{11}} = \frac{10(3\sqrt{3}-4)}{11}$

what about Max. Value?

If $y = 0$ → No triangle



$4x = 10 \quad x = 2.5$

Area $= (2.5)^2 = \boxed{6.25}$ ✓max

If $x = 0$ → No Square

$6y = 10 \quad y = \frac{5}{3}$

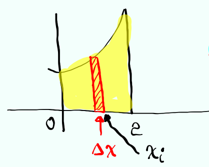
Area $= \left(\frac{5}{3}\right)^2 \cdot \sqrt{3} = \frac{25\sqrt{3}}{9}$

$\approx \boxed{4.8}$

Nov 13-7:46 AM

Find the area below $f(x) = 4 + x^2$, above x-axis from $x = 0$ to $x = 2$.

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$



$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

$x_i = a + i\Delta x$

$= 0 + i \cdot \frac{2}{n} = \frac{2i}{n}$

$f(x_i) = 4 + \left(\frac{2i}{n}\right)^2 = 4 + \frac{4i^2}{n^2}$

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left[4 + \frac{4i^2}{n^2}\right] \cdot \frac{2}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{8}{n} + \frac{8i^2}{n^3}\right)$$

$$= \lim_{n \to \infty} \left[\sum_{i=1}^{n} \frac{8}{n} + \sum_{i=1}^{n} \frac{8i^2}{n^3}\right]$$

$$= \lim_{n \to \infty} \left[\frac{8}{n} \cdot \sum_{i=1}^{n} 1 + \frac{8}{n^3} \cdot \boxed{\sum_{i=1}^{n} i^2}\right]$$

$$= \lim_{n \to \infty} \left[\frac{8}{\cancel{n}} \cdot \cancel{n} + \frac{\cancel{8}^{4}}{n^3} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}_{3}}\right]$$

$$= \lim_{n \to \infty} \left[8 + \frac{8n^3 + \cdots}{3n^3}\right]$$

$$= 8 + \frac{8}{3} = \boxed{\frac{32}{3}}$$

Nov 14-8:15 AM

Given $f'(x) = 4 + x^2$ $\quad f(0) = 0$

Find $f(x)$ $\quad f(x) = 4x + \frac{1}{3}x^3 + C$

$f(0) = 4(0) + \frac{1}{3}(0)^3 + C = 0$

$\boxed{C = 0}$

$f(x) = 4x + \frac{1}{3}x^3$

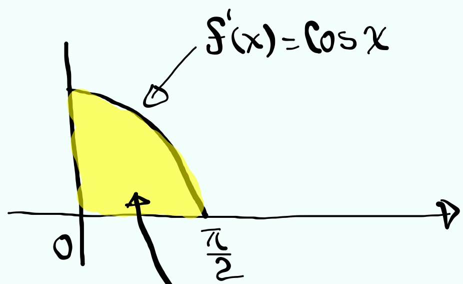
Evaluate $f(2) - f(0)$

$= 4(2) + \frac{1}{3}(2)^3 - 0 = 8 + \frac{8}{3} = \boxed{\frac{32}{3}}$

Nov 13-8:30 AM

The area below $f'(x) \geq 0$, above x-axis from $x = a$ to $x = b$ is $f(b) - f(a)$.

$f'(x) = \cos x$



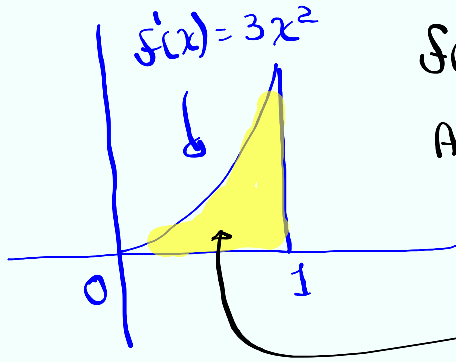
$f(x) = \sin x + C$

$f(\frac{\pi}{2}) - f(0) = (\sin\frac{\pi}{2} + C) - (\sin 0 + C)$

$= 1 + \cancel{C} - 0 - \cancel{C}$

$= \boxed{1}$

Nov 14-8:29 AM



$f(x) = x^3 + C$

Area $= f(1) - f(0)$

$= (1^3 + C) - (0^3 + C)$

$= \boxed{1}$

Verify this by using

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$f(x) = 3x^2$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_i = a + i\Delta x$

$= i \cdot \frac{1}{n}$

Nov 14-8:33 AM